**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:** Since work begins 10mins after the car is dropped, the time left to complete work is 50mins.

Probability that service manager cannot meet his commitment=P(X>50) = 1-Pr(x<=50)

X is the time taken to complete work.

Now convert 50 to z-score.

Standard normal variable Z = (x-µ)/ 𝜎

=( x-45)/8

P(X<=50)=P(Z<=(50-45)/8)=PR(Z<=0.625)=0.73237

**=73.237%(in z-table)**

Probability that service manager will not meet his commitment is:

= 100 - 73.237

**=0.2676**

**Therefore the answer is B.**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Ans:** µ = 38 and 𝜎 = 6

probability of employees > 44 – Pr(x>44) = 1=Pr(x<=44)

Z=(X- µ)/ 𝜎

=(x-38)/6

Pr(x<=44)=Pr(z<=(44-38)/6)=Pr(Z<=1)

**= 0.84134**

**= 84.134%**

Probability that employees will be greater than 44

**=100-84.134**

**=15.866**

Probability that employees between 38 and 44

=Pr(x<=44)-Pr(x>=38)

Here,Pr(x<=44)=0.84134

Pr(x>=38)=Pr(z>=(38-38)/6)=Pr(z>0)

=0.5

Therefor, Pr(x<=44)-Pr(x>=38)

**=0.84134-0.5**

**=0.34134**

**=34.134%**

Hence the statement “more employees at the processing center are older than 44 than between 38 and 44” is True

B.) A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:** Probability of employees less than 30=Pr(X<30)

Z=(X- µ)/ 𝜎

=(30-38)/6

Pr(x<30)=Pr(z<=(30-38)/6)=Pr(Z<=1.3333)

**=0.9176**

**=9.17%**

So,the number of emplyoees with probability 0.0917 of them being under 30

=400\*0.0917

**=36.68**

**=36**

Hence the statement “training program for emplyoees under the age of 30 at the center would be expected to attract about 36 employees” is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:**As we know that if X ~ N(μ, σ²) and Y ~ N(μ2, σ2²) are two independent Random variables than,

X + Y ~ N(μ1 + μ2, σ1²+ σ2²) and X - Y ~ N(μ1 - μ2, σ1²+ σ2²)

Similarly if Z = ax +by,

Where X and Y are as defned above that is z is linear combinaton of X and Y then Z ~ N(aμ1 + bμ2 a² σ1² + b² σ2² )

therefore in the queston

2X1~ N(2µ,4 σ²) and

X1 +X2 ~ N(µ + µ, σ²+ σ²) ~ N(2µ , 2σ²)

**2X1 – (X1 +X2) = N(4µ ,6 σ²)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:**

The probability of getting value between a and b is 0.99

So, the probability of getting value outside a and b is 1 – 0.99 = 0.01.

The probability towards left of a = -0.01/2 = -0.05

The probability towards right of b = 0.01/2 = 0.05

As we have probabilities of a and b, we need to calculate the probability of X(the random variable at a and b which has these probabilities.

Now find z to calculate X

Z=(X- μ)/ σ

For probability of 0.005,z values is-2.57

Z\* σ+ μ=x

-(-2.57)\*20+100=151.4

**(-2.57)\*20+100=48.6**

Therefore option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans:** **Rs. 603.68**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans: 476.33**

C. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:** **First division.The division that has a larger probability of making a loss in a given year is the first division**